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The solid angle subtended by a well-type detector and a cylindrical source

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H I G H L I G H T S

- ▸ Evaluation of solid angle subtended between a well-type detector and a cylindrical radioactive source.
- ▸ A Monte Carlo approach, based on total variance reduction, developed for the evaluation.
- ▸ Evaluations within 0.7% of those in literature.
- ▸ Effect of self-absorption, within the source, on the geometrical efficiency was investigated.
- ▸ Self-absorption significant for gamma-ray energies below 300 keV regardless the source matrix and radius.

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A B S T R A C T

A Monte Carlo approach, based on total variance reduction, was presented in order to evaluate the solid angle subtended between a well-type NaI detector and a cylindrical source within the well. The results obtained, in the form of the geometrical efficiency ε_g, were within 0.7% of those in the literature, for a point and a volumetric radioactive source within the well. The effect of self-absorption on the geometrical efficiency was investigated for different gamma-ray energies emitted by the volumetric source, different source matrices and radii. Self-absorption is found to be particularly significant for gamma-ray energies below 300 keV regardless the source matrix and radius. Furthermore, self-absorption becomes significant, in the case of the larger radius volumetric source, for the gamma-ray energies up to 1 MeV considered. Hence, the effect of self-absorption on the geometrical factor should be considered in the absolute quantification of radioactivity of volumetric sources.

1. Introduction

Knowledge of the solid angle Ω, subtended by a detector and a 3D radioactive source, is essential in the absolute measurement of the radioactivity of the source. In this context, both analytical and Monte Carlo approaches have been used to evaluate the solid angle for different detector-sample geometries. Although the former is straightforward in the case of a point source, the case of 3D sources is tackled with the Monte Carlo approach (Gardner and Verghese, 1971; Holmberg and Rieppo, 1973).

In this work, a Monte Carlo approach is presented to evaluate the solid angle subtended between a well-type NaI detector and a cylindrical source within the well. The method is based on total variance reduction (Wielopolki, 1977; Nicolaou et al., 1987, 2006). The results obtained, in the form of the geometrical efficiency ε_g, are evaluated against the literature. Furthermore, the effect on ε_g of the self-absorption within the 3D sample of the emitted gamma-rays is studied in the case of different detectors and samples.

2. Materials and methods

An isotropic emission of gamma-rays from radioactivity uniformly distributed within a cylindrical source, placed within the detector well, is considered. The source, co-axially placed within the detector well, has a radius and length of RS and SL, respectively, while its circular base is at a distance H_o above the bottom of the well (Fig. 1, side view). A random number generator is used to generate: (1) points within the source where disintegration occurs; (2) the random directions of the gamma-rays subsequently emitted from these points. Then, the variable position (H, R, β) of a generated point P is

$$H = H_o + (SL \times X1) \quad (1)$$

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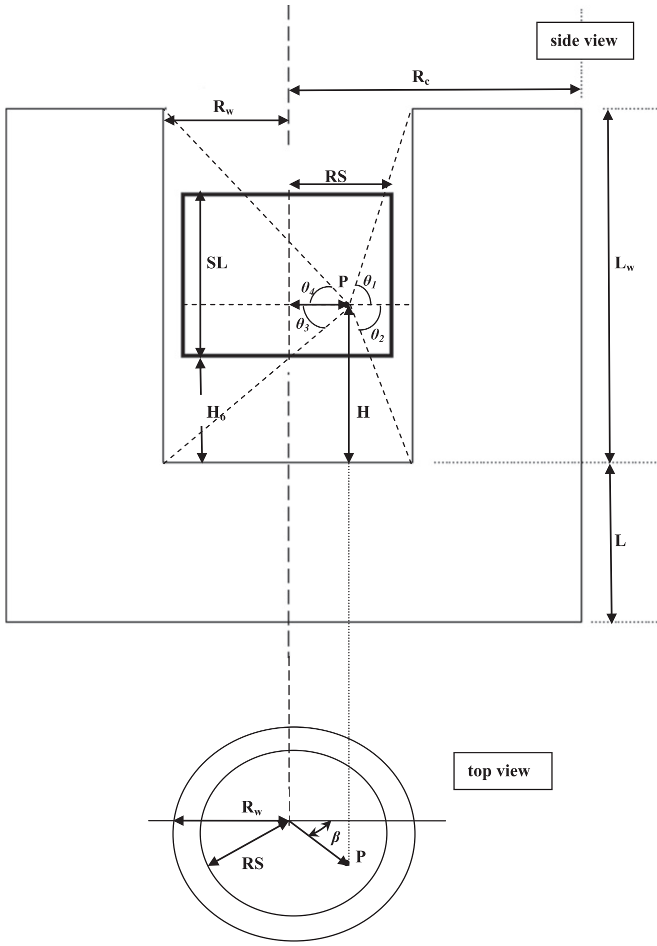

Fig. 1. Side and top views of the co-axial cylindrical well detector-source geometrical configuration considered in the study.

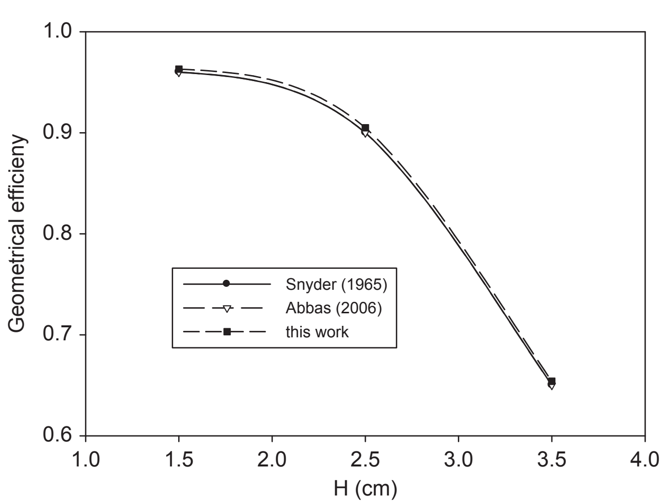

Fig. 2. Variation of the geometrical efficiency of a NaI well-type detector for different positions of an axial point source from the bottom of the crystal well.

$$R = RS \times X2 \tag{2}$$

$$\beta = 2\pi \times X3 \tag{3}$$

where β is a horizontal angle of P (Fig. 2, top view) and $X1$, $X2$ and $X3$ are three independent random numbers, each one equidistributed in the range [0, 1].

In the case of an isotropic emission at point P into a sphere of unit radius, the solid angle $d\Omega$ in spherical coordinates is

$$d\Omega = \sin\theta \times d\theta \times d\alpha \tag{4}$$

where θ and α are the longitudinal and horizontal angles, respectively. The joint probability density distribution $P(\theta, \alpha)$ for the isotropic gamma-ray emission by the point, describing the fractional radiation emitted in $d\Omega$ (Wielopolki, 1977), is

$$P(\theta, \alpha) \times d\theta \times d\alpha = \frac{d\Omega}{4\pi} \tag{5}$$

yielding

$$P(\theta) = \tfrac{\sin\theta}{2} \quad 0 \le \theta \le \pi, \tag{6}$$

$$P(\alpha) = \tfrac{1}{2\pi} \quad 0 \le \alpha \le 2\pi. \tag{7}$$

Once the point P is generated within the source, the random direction (α, θ) of the gamma-rays subsequently emitted from this point is generated. The selection of the longitudinal angle θ may vary between the angles $[\theta_1,\theta_2]$, $[\theta_2,\theta_3]$, $[\theta_3,\theta_4]$, where

$$\theta_1 = \arctan\left((L_w - H)/Z\right) \tag{8}$$

$$\theta_2 = \arctan\left(H/Z\right) \tag{9}$$

$$\theta_3 = \arctan\left(H/Y\right) \tag{10}$$

$$\theta_4 = \arctan\left((L_w - H)/Y\right) \tag{11}$$

and

$$(Z \times \cos\alpha + R \times \cos\beta)^2 + (Z \times \sin\alpha + R \times \sin\beta)^2 = R_w^2 \tag{12}$$

$$(Y \times \cos\alpha - R \times \cos\beta)^2 + (Y \times \sin\alpha + R \times \sin\beta)^2 = R_w^2 \tag{13}$$

with Z and Y been the distances of point P from the intersection of the random direction (α, θ) with the detector surface.

The random directions of the gamma-ray emission at point P are chosen through the Eqs. (6) and (7) under the restriction that they will intersect the detector. For a random direction (α, θ), such a restriction is associated with a total weighting factor W_i

$$W_i = W(\theta) \times W(\alpha) \tag{14}$$

$$W(\alpha) = \left[\int_{-\alpha_{\max}}^{\alpha_{\max}} P(\alpha) \times d\alpha\right] \Big/ \left[\int_0^{2\pi} P(\alpha) \times d\alpha\right] \tag{15}$$

$$W(\theta) = \left[\int_{\theta_{\min}}^{\theta_{\max}} P(\theta) \times d\theta\right] \Big/ \left[\int_0^{\pi} P(\theta) \times d\theta\right] \tag{16}$$

where $W(\theta)$ and $W(\alpha)$ are weighting factors associated with the particular selection on the angles θ and α. The factor W_i represents the solid angle subtended by the detector from point P. Then, the solid angle Ω for the configuration comprising the detector and the cylindrical source is

$$\Omega = \left(\frac{4\pi}{N}\right) \times \sum_{i=1}^{N} W_i \tag{17}$$

where N is the number of random point positions and gamma-ray directions sampled. Hence, the geometrical efficiency ε_g would be $(\Omega/4\pi)$, effectively describing the fraction of the gamma-rays emitted by the source that enter the detector. The solid angle Ω would take values ranging up to 4π, with the geometrical efficiency ε_g reaching the value of 1 in the case of a 4π geometry.

In the case of dense samples and gamma-rays of low energy, the effect of self-absorption within the sample may be appreciable. In this case, the number of gamma-rays emerging from the sample is reduced by the factor $F_{att} = \exp(-\mu \times l)$, where μ is the linear attenuation coefficient of the material of the sample at the gamma-ray energy considered and l is the path length of the particular simulated gamma-ray direction within the sample. Along the random directions (α; θ_1,θ_2) and (α; θ_3,θ_4), l is given by the distances $l1$ and $l2$, respectively, where

$$(l1 \times \cos\alpha + R \times \cos\beta)^2 + (l1 \times \sin\alpha + R \times \sin\beta)^2 = RS^2 \tag{18}$$

$$(l2 \times \cos\alpha - R \times \cos\beta)^2 + (l2 \times \sin\alpha + R \times \sin\beta)^2 = RS^2 \tag{19}$$

Self-absorption would affect the shape of the sample, thus altering the solid angle Ω it subtends with the detector and given by Eq. (17). In this case, Ω is reduced to an effective solid angle Ω_{eff}

$$\Omega_{eff} = \left(\frac{4\pi}{N}\right) \times \sum_{i=1}^{N} (F_{\text{att}} \times W_i) \tag{20}$$

3. Results

The geometrical efficiency calculated in the present work is evaluated against results from the literature (Snyder, 1965; Holmberg and Rieppo, 1973; Abbas, 2006). In this context, the geometry shown in Fig. 1 has been simulated for the case of an isotropically radiating point source which is axially placed within the well of the detector. The source is at distances (H) of 1.5, 2.5 and 3.5 cm above the bottom of the well. The dimensions of the detector involved are given in Table 1 (case A). The variation of the geometrical efficiency for the point source placed in different positions along the axis of the well is shown in Fig. 2. The deviation of the values from this study is within 0.5% from those found in the literature.

Furthermore, the geometry shown in Fig. 1 has been simulated for the case of an isotropically radiating cylindrical source symmetrically placed at different positions along the axis of the well. It is assumed that there is no self-absorption within the source of the gamma-rays emitted by the source. The cylindrical source, with both a height SL and diameter RS of 1 cm, is placed at a distance (H_o) of 0, 0.5 and 1 cm above the bottom of the well. The dimensions of the well-type detector involved are given in Table 1 (case B). The variation of the geometrical efficiency for the cylindrical source placed in different positions along the axis of the well is shown in Fig. 3. The deviation of the values obtained from this study is within 0.7% from those found in the literature (Abbas, 2006).

In the case of a volumetric radioactive source, such as the one considered in Fig.1, self-absorption of the gamma-rays emitted may occur. The absorption problem has no simple solution as the attenuation of gamma rays depends on many parameters such as gamma-ray energy, sample composition, sample density, and sample-detector geometry. In the case of considerable self-absorption within a volumetric sample with a uniform radio-activity distribution, an inner part of the sample could become almost 'invisible' with its emitted gamma-rays been absorbed prior to their exit from the sample.

The effect of self-absorption on the geometrical efficiency, for a radioactive source and a well-type detector arrangement, has

Table 1
Parameters (cm) of the simulated well-type detectors.

	Case A	Case B	Case C
Radius of the crystal well (R_w)	0.9575	1.05	5.5
Length of crystal well (L_w)	3.81	3.6	10
Crystal radius (R_c)	2.2225	2.65	10
Crystal length (L_w+L)	5.08	5.5	20

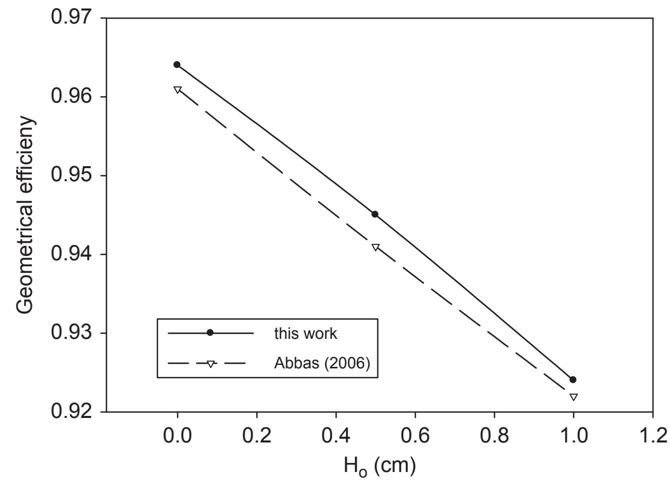

Fig. 3. Variation of the geometrical efficiency of a NaI well-type detector for different heights (H_o) of a cylindrical source from the bottom of the crystal well.

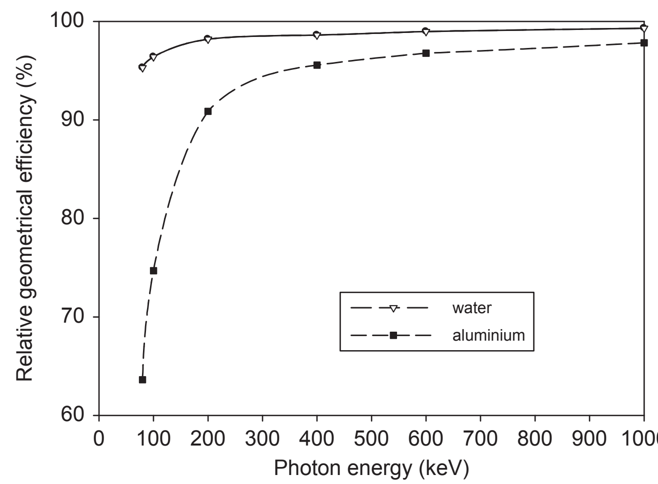

Fig. 4. The effect of self-absorption on the geometrical efficiency for source radius RS=5 cm.

been simulated for different source dimensions, matrix and gamma-ray energy emitted by the source. The NaI well-type detector of Case C (Table 1) has been considered. An isotropically radiating cylindrical source is considered, which is symmetrically placed along the axis of the well at a distance (H_o) of 1 cm above the bottom of the well. The cylindrical source has a height SL of 5 cm, while two different diameters (RS) of 1 and 5 cm are considered. Three different matrices of the source are considered, namely void, water and aluminum, with the latter two giving rise to self-absorption.

Self-absorption would effectively render a non-uniform radio-activity distribution within a source. This is evident on the basis of the half-value layer of, for example gamma-rays of 200 keV within a water, soil and aluminum matrices. The values of 5, 3.3 and 2 cm, respectively, indicate that for the larger diameter source of 5 cm, a considerable attenuation within the sample would occur even in the case of the water matrix. Hence, a volumetric source, with radioactivity distribution different from the one without self-absorption, is effectively created.

The effect of self-absorption on the geometrical efficiency has been studied as a function of the gamma-ray energy emitted by the source for the three different source matrices. The effect is shown for the water and aluminum matrices relative to the void matrix without self-absorption effects, for two different

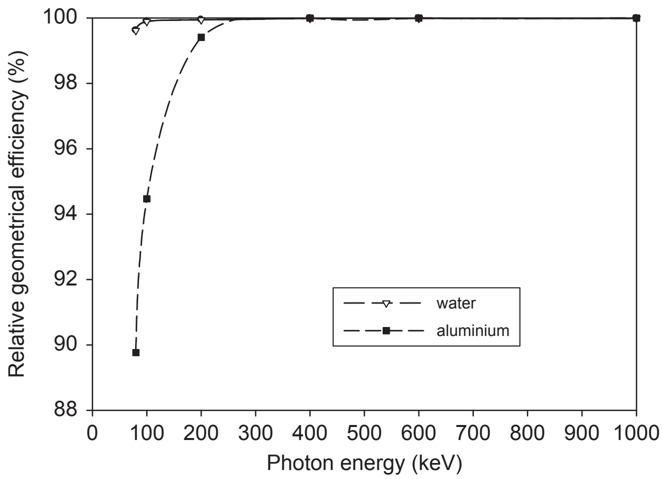

Fig. 5. The effect of self-absorption on the geometrical efficiency for source radius $RS = 1$ cm.

source radii (*RS*) of 5 and 1 cm, respectively (Figs. 4 and 5). Self-absorption is particularly considerable for energies below 300 keV regardless the source matrix and radius. Nevertheless, self-absorption is significant throughout the gamma-ray energy range considered for the larger radius volumetric source. Hence, the effect of self-absorption on the geometrical factor should be considered when quantification of radioactivity is considered.

4. Conclusions

Self-absorption within a volumetric radioactive source, of the gamma-rays emitted by the source, is an important issue in absolute quantification of this radioactivity. In this sense, a Monte Carlo approach, based on total variance reduction, was presented in order to evaluate the solid angle subtended between a well-type NaI detector and a cylindrical source within the well. The results obtained, in the form of the geometrical efficiency ε_g, were found to be within 0.7% of those in the literature, in the case of a point and a volumetric radioactive sources within the well.

The effect of self-absorption on the geometrical efficiency was investigated as a function of the gamma-ray energy emitted by the source for different source matrices and two source radii. Self-absorption is particularly significant for gamma-ray energies below 300 keV regardless the source matrix and radius. Furthermore, self-absorption is significant, in the case of the larger radius volumetric source, for the gamma-ray energies up to 1 MeV considered. The effect of self-absorption on the geometrical factor should therefore be considered in the absolute quantification of radioactivity of volumetric sources.

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